Exercise 0. Do the verifications/exercises given during the lecture.

Exercise 1. (i) Let $G$ be a topological group and $H < G$ a subgroup. Prove that the closure $\overline{H}$ of $H$ in $G$ is also a subgroup. Same question for normal subgroups.

(ii) According to (i), we can in particular consider the quotient group $G/\{e\}$. Prove that the quotient topology makes this a Hausdorff topological group.

Exercise 2. Find a Hausdorff group topology on $\mathbb{R}$ which is not the usual neither the discrete topology.

Exercise 3. Prove that $\text{SL}_2(\mathbb{R})$ and $\text{SO}(2)$ are connected (when endowed with the topology coming from $\mathbb{R}$).

Then generalize to $n \times n$ matrices and consider also $\text{GL}_n(\mathbb{C})$.

Hints: there are many ways to approach this, but it is useful to think about what we know from linear algebra. Recall e.g. that $\text{SO}(n)$ denotes the group of all orthogonal matrices with determinant one. It is also good to remember that the image of a connected space under a continuous map is connected (verify this assertion). What happens with $\text{O}(n)$ and with $\text{GL}_n(\mathbb{R})$?