Exercise 0. Do the verifications/exercises given during the lecture.

Exercise 1. Let $G$ be a group endowed with a topology. Prove that $G$ is a topological group if (and only if) the map $G^2 \to G$ defined by $(x, y) \mapsto xy^{-1}$ is continuous.

Exercise 2. Give an example of a group $G$ and of two closed subsets $A, B \subseteq G$ such that $AB$ is not closed.
Hint: for $G$, you can take $\mathbb{R}^2$ or even $\mathbb{R}$.

Exercise 3. Choose an identification between $\mathbb{R}^4$ and the space of all $2 \times 2$-matrices. Prove that the group $G = \text{GL}_2(\mathbb{R})$ is a topological group for the induced topology.
Write down your proof carefully and in detail, so that you can adapt it to $\text{GL}_n(\mathbb{R})$ with $n \in \mathbb{N}$.

Exercise 4. Consider the space $\mathbb{N}^\mathbb{N}$ of all maps $\mathbb{N} \to \mathbb{N}$ with the topology of pointwise convergence. Prove that the group $\text{Bij}(\mathbb{N})$ of all bijections is not a closed subset of $\mathbb{N}^\mathbb{N}$.