Some key concepts introduced so far:

Group topologies and $\mathcal{N}_e$, the neighbourhoods of $e$

The quotient map $G \to G/H$ for $H < G$

Separation properties (i.e. Hausdorff etc.)

Connectedness

$\text{GL}_d(\mathbb{R})$ and some of its properties

The topology of pointwise convergence on bijections

The neutral component $G^\circ$ and the group of components $G/G^\circ$

Totally disconnected and 0-dimensional groups

Residually finite groups

van Dantzig’s theorem

Projective limits and profinite groups

To have (or not to have) small subgroups

Haar measure on compact groups

Fixed points for compact groups

Uniform continuity on topological groups

Haar measure for locally compact groups