Exercise 1
Consider a tree $X$ (connected graph without circuit) such that every vertex has three neighbours. In particular, this tree is infinite. Draw (a piece of) $X$; it is very similar to the tree of Problem Set 9. Remind yourself why we can consider $X$ as a CAT(0) space. Describe $\partial X$.

(i) Give an example of an elliptic isometry of $X$. You can describe it in words (but precisely, please) rather than formulas.

(ii) Same question for a hyperbolic isometry.

(iii) Prove that every hyperbolic isometry has a unique axis.

(iv) Prove that $X$ does not admit any parabolic isometry.

Exercise 2
Given two sets $A$ and $B$, we define the “join”

$$A \ast B = A \times B \times [0, \pi/2] / \sim,$$

where $\sim$ is the equivalence relation identifying $(a, b, 0)$ with $(a', b, 0)$ for all $a, a' \in A, b \in B$ and identifying $(a, b, \pi/2)$ with $(a, b', \pi/2)$ for all $a \in A, b, b' \in B$.

Given CAT(0) spaces $X$ and $Y$, describe a bijection of $\partial (X \times Y)$ with $\partial X \ast \partial Y$.

Remark: the particular case of $X = \mathbb{R}^{n+1}$ and $Y = \mathbb{R}^{m+1}$ gives $S^n \ast S^m \simeq S^{n+m+1}$. Make sure you agree with the latter fact about spheres!