Analysis on groups

Problem Set 9 11 November 2020

**Only Exercise**

Let $G$ be a countable amenable group. Prove that there is a probability measure $\mu$ on $G$ such that for all $g \in G$ we have

$$\lim_{n \to \infty} \|g\mu^n - \mu^n\| = 0$$

and also

$$\lim_{n \to \infty} \|\mu^n g - \mu^n\| = 0.$$ 

To do this, prove that there is $\mu$ such that $\mu = \hat{\mu}$ and the first limit holds, recalling that $\hat{\mu}(g) = \mu(g^{-1})$. (Why is this sufficient?)

*This exercise is just an excuse to make you go through all the details of the theorem proved in class last week. Thus, make sure that you justify in detail every step and every inequality! You are not allowed to write “... and then we proceed like in class...”*

For instance, we used the following fact — verify it carefully:

Let $\alpha$ be $(S, \epsilon)$-invariant. Then $\|\vartheta + \alpha - \alpha\|_1 < \epsilon$ for any probability $\vartheta$ supported on $S$. 