Exercise 1
Let $Q$ be a group acting by automorphisms on a group $N$ and consider the corresponding semi-direct product $G = N \rtimes Q$.
(i) Since $N \triangleleft G$ is a normal subgroup, $G$ acts by conjugation on $N$. Work out explicitly the relation between this action and the initial action of $Q$ on $N$.
(ii) Consider the special case where $Q = \text{SL}_2(\mathbb{R})$ acts by multiplication on $N = \mathbb{Z}^2$. Is $Q$ co-amenable in $G$?

Exercise 2
Let $X$ be an amenable $G$-set and $Y$ be an amenable $H$-set. Prove that the action of $G \times H$ on $X \times Y$ is amenable.

Exercise 3
The action of a group $G$ on a set $X$ is called paradoxical if there is a partition
$$X = A_1 \sqcup \ldots \sqcup A_n \sqcup B_1 \sqcup \ldots \sqcup B_m$$
and group elements $g_1, \ldots, g_n, h_1, \ldots, h_m$ which lead to two new partitions
$$X = g_1 A_1 \sqcup \ldots \sqcup g_n A_n \quad \text{and} \quad X = h_1 B_1 \sqcup \ldots \sqcup h_m B_m.$$  
(i) Check that this is impossible if $n + m < 4$.
(ii) Suppose that $n + m = 4$; prove that $G$ contains a subgroup isomorphic to $F_2$.  

Analysis on groups