Exercise 0
Let $G$ be an amenable group. Prove that there is $\mu \in \nu(G)$ which is invariant under both the left and the right multiplication.

Exercise 1
Let $G$ be a group. Prove that $G$ contains a maximal normal amenable subgroup, and that it is unique. We call it the amenable radical of $G$ and denote it by Ramen($G$). Show that the amenable radical of $G$/Ramen($G$) is trivial.

Exercise 2
Find a sequence $G_n$ of amenable groups such that the product $\prod_n G_n$ is non-amenable.

Hint: By Ex. 2 in Problem Set 3, the group $\text{SL}_2(\mathbb{Z})$ is non-amenable.

Exercise 3
Make sure that you understand the definition of $\bigoplus_{i \in I} G_i$ for a family $\{G_i\}_{i \in I}$ of groups $G_i$. Prove that $\bigoplus_{i \in I} G_i$ is amenable if all of the $G_i$ are amenable. Compare to Ex. 2 and meditate.

Exercise 4
(i) Let $G$ be a finite group. Prove that $G^N := \prod_{n \in \mathbb{N}} G$ is amenable.
(Compare again to Ex. 2.)

(ii) Give an example of an amenable group $G$ such that $G^N$ is non-amenable.

Hint: Use both Ex. 2 and Ex. 3.

Other Exercise
We have emphasized the difference between a group being a directed union of subgroups — and being simply a union of subgroups.

Can a group $G$ be the union of two subgroups $G_1, G_2 < G$? (Of course, we assume $G_i \neq G$.) How about three?