Exercise 0
Check that the entire proof of the Banach–Tarski theorem holds more generally in $\mathbb{R}^n$ for all $n \geq 3$. What goes wrong for $n = 2$?

Exercise 1
The Banach–Tarski theorem states that the closed unit ball $B \subseteq \mathbb{R}^3$ is equidecomposable with every subset $A \subseteq \mathbb{R}^3$ that is bounded and of non-empty interior.

(i) Give a precise proof that this fails for every $A$ that is not bounded.

(ii) Give an example to show that this fails for some bounded $A$ with empty interior.

(iii) Give an example to show that this holds for some $A$ with empty interior.

Exercise 2
Let $G = \text{Isom}(\mathbb{R}^4)$. Let $B_3 \subseteq \mathbb{R}^4$ be the closed unit ball of $\mathbb{R}^3$ viewed as a subset of $\mathbb{R}^4 = \mathbb{R}^3 \oplus \mathbb{R}$; thus $B_3$ has empty interior.

(i) Explain why it is still true that $B_3$ is $G$-equidecomposable to two disjoint copies of $B_3$ in $\mathbb{R}^4$.

(ii) Is $B_3$ $G$-equidecomposable to the unit ball $B_4$ of $\mathbb{R}^4$?