Analysis on groups

Problem Set 9 18 April 2019

Exercise 1
Suppose that there is a piecewise-G surjection $A \to B$. Prove that there is a piecewise-G injection $B \to A$.
Show that the converse does not hold.

Exercise 2
Consider the notation

$$X = A_1 \sqcup \ldots \sqcup A_n \sqcup B_1 \sqcup \ldots \sqcup B_m = g_1 A_1 \sqcup \ldots \sqcup g_n A_n = h_1 B_1 \sqcup \ldots \sqcup h_m B_m$$

used in class for a paradoxical decomposition. The number $n + m$ is called the **Tarski number** of this decomposition.

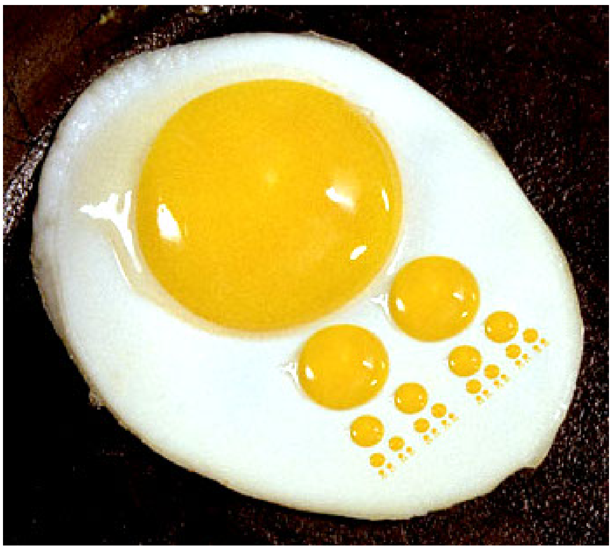
i) Show that the Tarski number is ≥ 4.

ii) If the Tarski number is 4, prove that the group contains a free subgroup F_2.
Hint: show first that you can suppose $g_1 = h_1 = e$.

Exercise 3
We denote by SO(3) the group of 3×3 orthogonal (real) matrices with determinant one. We denote by SU(2) the group of 2×2 unitary (complex) matrices with determinant one.

i) Make sure that you understand these definitions and the fact that SO(3) is the group of rotations around the origin in $\mathbf{R}^3$.

ii) Understand the statement and the proof of the fact that there is a surjective homomorphism $\mathrm{SU}(2) \to \mathrm{SO}(3)$. What is the kernel?
Hint for finding the homomorphism: try to use complex numbers to describe real vectors.



. . . The EGG team wishes you a paradoxical Easter break!