

Analysis on groups

Problem Set 9

27 April 2017

Exercise 1

The goal of this exercise is to illustrate that the “twin marriage” technique cannot be replaced by the more naive “successive marriage” approach.

Find a concrete example of a finite selection problem $\Sigma: X \rightarrow \mathcal{P}_f(Y)$ such that $|\Sigma_A| \geq 2|A|$ for all $A \subseteq_f X$ and of an injective selection $\sigma: X \rightarrow Y$ such that the new selection problem

$$X \ni x \mapsto \Sigma(x) \setminus \sigma(X) \subseteq_f Y$$

does not admit an injective selection anymore.

Exercise 2

We are proving in class that the closed unit ball B of \mathbf{R}^3 is equidecomposable with any subset $A \subseteq \mathbf{R}^3$ that is (1) bounded and (2) with non-empty interior.

- (i) Check that condition (1) is necessary.
- (ii) Find an example showing that condition (2) is not necessary.

Exercise 3

Let G be a group acting on a set X .

(i) Check that the action on X is amenable as soon as there exists some orbit $Gx_0 \subseteq X$ on which the G -action is amenable.

**(ii)* Give an example of an amenable G -action such that the action on every orbit is non-amenable.