Exercise 1
Let $G$ be a group acting on a set $X$. Prove that the following condition is equivalent to $(R_2)$:
For every $S \subseteq G$ and every $\varepsilon > 0$ there is $v \in \ell^2(X)$ with
$$\left\| \frac{1}{|S|} \sum_{s \in S} sv \right\|_2 > (1 - \varepsilon)\|v\|_2.$$ (*Meditate about the corresponding statement for $(R_p)$, especially when $p = 1$.)

Exercise 2
Let $G = \{ x \mapsto ax + b : a \in \mathbb{R}^*, b \in \mathbb{R} \}$ be the group of affine transformations of $\mathbb{R}$. Consider the $G$-action on $X = \mathbb{R}$.
(i) Find a Følner set $A$ for $\varepsilon = 1/10$ and $S = \{+1, 2\}$.
(ii) Prove that $\mathbb{Z} \subseteq \mathbb{R}$ contains no such Følner set $A$.

Exercise 3
Prove that the Reiter property $(R_1)$ implies the Følner property (F).

This follows from the Tarski theorem proved later, but find a direct proof.

Exercise 4*
Let $G$ be a group with an amenable action on a set $X$. Prove that the Reiter property $(R_1)$ holds.

Again, this follows from the Tarski theorem, but try to find a direct proof.
The first step is to show that there is a net $(\varphi_j)_{j \in J}$ in $\ell^1(X)$ with $\|\varphi_j\|_1 = 1$ and such that for all $g \in G$ the net $g\varphi_j - \varphi_j$ tends to zero weakly.