Analysis on groups

Problem Set 5 22 March 2018

Exercise 0
Make sure that you understood the definition of $\bigoplus_{i\in I} G_i$ for a family $\{G_i\}_{i\in I}$ of groups G_i. Prove that $\bigoplus_{i\in I} G_i$ is amenable if all of the G_i are amenable. Compare to Ex. 4 in Set 4.

Exercise 1
(i) Let G be a finite group. Prove that $G^{\mathbf{N}} := \prod_{n\in\mathbf{N}} G$ is amenable.
(Compare again to Ex. 4 in Set 4.)

(ii) Give an example of an amenable group G such that $G^{\mathbf{N}}$ is non-amenable.

Exercise 2
Let G be a group. Prove that G contains a *maximal* normal amenable subgroup, and that it is unique. We call it the *amenable radical* of G and denote it by $\mathrm{Ramen}(G)$. Show that the amenable radical of $G/\mathrm{Ramen}(G)$ is trivial.

Exercise 3
(i) If needed, learn what a *simple* group is and what the *alternating* group A_n is.

(ii) Find out the truth about the simplicity of A_n — using the library, wikipedia or your brain (or a combination of the above).

(iii) Define A_∞ as the increasing union of A_n as $n \to \infty$; this might require some thinking.

(iv) Prove that A_∞ is an infinite simple amenable group.