Analysis on groups

Problem Set 2 2 March 2017

Exercise 1
Prove that the group $\mathbf{Z}^2$ is amenable.

This will later be a very special case of a theorem, but for now proceed as follows: construct a sequence of means $\{\mu_n\}$ in $\mathscr{M}(\mathbf{Z}^2)$ such that any accumulation point of this sequence is invariant; try to imitate the case of $\mathbf{Z}$ seen in class.

Exercise 2
Prove that the sequence $\{\mu_n\}$ defined by

$$\mu_n = \frac{1}{n}\sum_{j=1}^{n} \delta_j$$

has no convergent subsequence in the compact space $\mathscr{M}(\mathbf{Z})$.

Exercise 3
Read `http://en.wikipedia.org/wiki/Net_(mathematics)` and use the library to understand the concept of *nets*, also known as *generalized sequences*. Understand the connection to open and closed sets, to continuous maps, to compactness. In particular, make sure to understand sub-nets.

Meditate about the fact that the sequence of Exercise 2 must admit a convergent sub-net.

Observation: there are only 3 exercises on this set!
Corollary: take your time to do them all carefully.