Let $X$ be a set. 

*If needed, refresh your memory about $\beta X$ by looking at Problem Set 3 (2 October).*

**Exercise 1**
Consider the following map “$\hat{\ }$”

\[
\ell^\infty(X) \rightarrow C(\beta X) \\
f \mapsto \hat{f} \\
\hat{f}(x) := x(f)
\]

Prove that it is well-defined, bijective and isometric.

**Exercise 2**
Let $A \subseteq X$. Prove that the closure of $A$ in $\beta X$ is open.
Can you describe $\Gamma_A$?

**Exercise 3**
Recall, if needed with the library, what the usual (“Radon”) probability measures on a compact space are. Then Ex. 1 shows that $\mathcal{M}(X)$ can be identified with the (convex compact) space of probability measures on $\beta X$. Check this.

In particular, given $A \subseteq X$ and $\mu \in \mathcal{M}(X)$, explain the interpretation of $\mu(A)$ in terms of usual measures on $\beta X$ using Ex. 2.

Now explain why this does not contradict the fact that means are usually not countably additive.