Only Exercise
Let $G$ be a countable amenable group. Prove that there is a probability measure $\mu$ on $G$ such that $\mu = \bar{\mu}$ and
\[
\lim_{n \to \infty} \|g\mu^n - \mu^n\| = 0
\]
for all $g \in G$, recalling that $\bar{\mu}(g) = \mu(g^{-1})$.
(Notice that this implies also $\lim_{n \to \infty} \|\mu^n g - \mu^n\| = 0$.)

This exercise is just an excuse to make you go through all the details of the theorem proved in class this week. Thus, make sure that you justify in detail every step and every inequality! You are not allowed to write “... and then we proceed like in class...”

For instance, we used the following fact — verify it carefully:
Let $\alpha$ be $(S, \epsilon)$-invariant. Then $\|\vartheta * \alpha - \alpha\|_1 < \epsilon$ for any probability $\vartheta$ supported on $S$. 